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A Multi-Objective Mixed-Integer Model for Asymmetric Roadside Greenbelt Design in Arid Highway Corridors

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Abstract. We formulate roadside greenbelt design in arid highway corridors as a constrained multi-objective mixed-integer nonlinear program on a discretized cross-section consisting of a median and two verges. Activating a cell installs a cluster of one species, and a per-zone irrigation variable fixes the realized crown geometry. Three functionals are minimized: the daily sensible-heat release, the worst-case A-weighted level at residential receivers, and a roadside risk index that combines fixed-object hazard with aeolian shelter. The irrigation volume is bounded by an ε -constraint. We prove that the marginal risk response is non-monotone in lateral offset, with a unique interior minimizer beyond the clear zone; that the functionals select incompatible sides, so that every efficient design is transversely asymmetric; and that the scalarized problem is NP-hard. A converged evolutionary study on a 1 km corridor at 33.3°N recovers both structural predictions: the designs place 63–100% of the activated verge cells on the receiver side, that is, the side carrying the modeled residences.

Keywords. Multi-objective optimization, Mixed-integer nonlinear programming, Pareto front, Roadside safety, Spatial discretization.

MSC. 90C29; 90C11; 90C27.

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1 Introduction

Roadside vegetation along arterial highways in hot-arid regions is asked to do several incompatible things at once: lower the thermal load of the corridor [2, 21, 29], attenuate traffic noise reaching adjacent housing, intercept wind-blown dust, and do so without consuming irrigation water that is already scarce—while not itself becoming a lethal roadside obstacle. Design guidance treats these requirements sequentially and qualitatively, layer by layer and without a common objective: the *Roadside Design Guide* [4] fixes a recoverable width first and admits planting only outside it, leaving the thermal, acoustic and shelter functions to be considered afterwards and separately. The optimization literature, for its part, generally maximizes a single benefit over a homogeneous strip, treating safety as an exogenous setback rather than as an objective with its own spatial structure [12, 23, 28].

Three features of the physical problem are lost in that treatment. First, the cross-section is not a strip but a sequence of zones with different admissibility rules: the median is bounded on both sides by travel lanes and is governed by a bilateral clear zone, each verge by a unilateral one. Second, the contribution of a plant depends on which side of the carriageway it occupies, and the preferred side differs between objectives—acoustic attenuation accrues only on the receiver side, aeolian shelter only on the upwind side, and shading transfer depends on the sign of the solar azimuth relative to the road axis. Third, the safety contribution is not monotone in lateral offset: a tree close to the travel way is a rigid fixed object and among the most frequently struck hazards in run-off-road crashes [3, 4, 32], while the same tree placed further out is essentially unstrikeable yet still contributes to a shelterbelt that reduces dust-event exposure. The marginal effect changes sign.

This paper formalizes those three features. Our contributions are:

1. a cell-activation mixed-integer formulation on a three-zone cross-sectional domain, with binary species assignment and a continuous irrigation-sufficiency variable governing allometric crown response (Section 3);
2. closed-form objective functionals for corridor sensible-heat release, worst-case receiver noise, roadside risk, and irrigation demand, including a shadow-projection lemma that makes the road azimuth an explicit model parameter (Section 4);
3. three structural results—non-monotonicity of the lateral risk response with a unique interior minimizer, side-selection incompatibility forcing transverse asymmetry of every efficient design, and NP-hardness of the scalarized problem (Section 6);
4. a numerical study that recovers both structural predictions from an unbiased evolutionary search, together with convergence, cost, repair-rule and parameter-sensitivity diagnostics (Section 8).

The remainder of this paper is organized as follows. Section 2 reviews related work. Section 3 fixes the notation (Section 3.1), states the modeling assumptions (Section 3.2), and introduces the geometry of the cross-section, the species catalogue, and the decision variables (Sections 3.3–3.5), after which the program is stated in Section 3.6. Section 4 constructs the four objective functionals, and Section 5 states the constraints. Section 6 establishes the structural properties of the model, and Section 7 describes the solution method. Section 8 reports the numerical study: the instance (Section 8.1), convergence, front quality, and cost (Section 8.2), the trade-off structure (Section 8.3), the recovery of the structural predictions (Section 8.4), and two sensitivity analyzes (Sections 8.5 and 8.6). Section 9 discusses the findings, and Section 10 concludes the paper.

2 Related Work

We position the model against four streams.

Spatial multi-objective greening. Liu et al. [22] optimize green-infrastructure layout on an urban land-use grid with NSGA-II, and Dong et al. [12] extend the same machinery to an InVEST-coupled multifunctionality objective. These are the closest methodological analogues in print: both share our skeleton of a discretized domain, binary allocation and NSGA-II. They differ in every structural respect this paper is about. Their domain is an isotropic land-use raster, not a laterally ordered cross-section; they carry no clear-zone admissibility rule, no side labels, and no objective whose marginal response changes sign in space. Their efficient sets therefore admit no analogue of Propositions 1 and 2.

Roadside safety and clear zones. The empirical literature treats roadside trees as hazards to be removed inside a recoverable width [3, 4]. White [32] documents the institutional tension between clear-zone-driven removal and the benefits of roadside vegetation, and White and Meixler [33] supply the corresponding natural experiment, estimating the crash effect of a large-scale removal program by difference-in-differences. That study is the natural empirical test of the shape derived in Section 6: φ_s predicts that removal helps only for stems inside a species-dependent sign-change offset y_s^0 and harms beyond it. We are not aware of crash data resolved finely enough in lateral offset to confirm or refute the interior minimizer, and record it as an open falsification target rather than a validation.

Vegetation and traffic noise. Van Renterghem et al. [31], Fang and Ling [16] and Meng et al. [24] measure belts of limited depth and agree that foliage contributes little and the ground effect rather more; they are our calibration context for κ and $\bar{A}_g$.

Location theory. Abstractly the risk functional makes this a *semi-obnoxious* location problem, a structure introduced by Church and Garfinkel [8]: what is unusual here is that the un-

desirability is not monotone in distance, so that where the classical model pushes facilities as far away as possible, (19) has an interior optimum. The worst-case receiver level (14) is a minimax (center-type) criterion in the sense of Elzinga and Hearn [15], and asking which parameter values make a prescribed offset optimal is an inverse location problem in the sense of Burkard et al. [6], with condition (20) the datum such an analysis would perturb. Algorithmically, Proposition 3 and the non-smoothness of (14) rule out exact mixed-integer linear treatment at corridor scale, and deterministic global solvers for mixed-integer programs with trigonometric nonlinearities remain restricted to far smaller instances [27]; hence a population heuristic [11] assessed by hypervolume [34]. Finally, Chen et al. [7] give a chance-constrained simulation-optimization treatment of roadside device placement, the template for relaxing our expected-value safety accounting (A11).

3 Model Setting: Notation, Assumptions and Decision Variables

This section fixes the notation (Section 3.1), states the hypotheses on which the formulation rests (Section 3.2), and defines the geometry of the cross-section, the species catalogue, and the decision variables (Sections 3.3–3.5); the program itself is stated in Section 3.6. Tables 1 and 2 list every symbol with its units, so that the functionals of Section 4 and the constraints of Section 5 may be read independently of the surrounding prose. Symbols are introduced once and are not reused: in particular σ denotes a side of the carriageway throughout, and the sensible-heat partition coefficients carry the letter γ .

3.1 Sets, Variables and Derived Quantities

The decision vector has a binary part, which says what is planted where, and a continuous part, which says how much water each zone receives; every other quantity in the paper is a function of those two. Table 1 lists the sets, the decision variables and the derived quantities in the order in which they are constructed in Section 4.1, and Table 2 the fixed data of an instance.

Table 1: Sets, indices, decision variables, and derived quantities.

Symbol	Meaning	Units / range
<i>Sets and indices</i>		
$\mathcal{Q}$	cell set, $\mathcal{Q} = \mathcal{Q}_0 \cup \mathcal{Q}_- \cup \mathcal{Q}_+$	$ \mathcal{Q} = n_c$
$\mathcal{Q}_0, \mathcal{Q}_\pm$	cells of the median and of the two verges	—

Continued on the next page

Table 1 (continued)

Symbol	Meaning	Units / range
p, p'	cell index, $p \in \mathcal{Q}$, with center (x_p, y_p)	—
$z(p), \iota(p)$	zone label of cell p ; station containing cell p	$\{0, -, +\}$, $\{1, \dots, N\}$
σ	side of the carriageway; σ_R receiver side, σ_W upwind side	$\{+, -\}$
$\mathcal{S}, \mathcal{S}_{scr} \subseteq \mathcal{S}$	species catalogue; species admitted as glare screen	—
$s, s(p)$	species index; species assigned to an occupied cell p	—
$\mathcal{J}, j; \mathcal{R}, q$	incoherent line-source segments; residential receivers	—
$\mathcal{T}, t$	daylight interval of the design day and time	s
<i>Decision variables</i>		
$\xi_{p,s}$	1 if cell p carries species s , else 0	$\{0, 1\}$
α_z	irrigation sufficiency applied to zone z	$[\alpha_{\min}, 1]$
<i>Quantities derived from (ξ, α)</i>		
$u_p = \sum_s \xi_{p,s}$	occupancy indicator of cell p	$\{0, 1\}$
n_s	cluster cardinality, Eq. (2)	individuals
$\tilde{r}_p, \tilde{h}_p$	realized crown radius and height, Eq. (4)	m
y_p^{off}	lateral offset of cell p from the nearest travel-lane edge	m
$y_p^{sh}(t), a_{\perp,p}(t)$	shadow center and transverse half-extent, Lemma 1	m
$\mu_p(t)$	share of the shadow of cell p over the pavement	$[0, 1]$
$A^{sh}(t), A^c$	shaded pavement area at time t ; verge canopy cover	m ²
m_w, Λ	daily irrigation mass; transpiration sufficiency index	kg d ⁻¹ , $[0, 1]$
$\varepsilon_p, \ell_{qj,p}$	extinction density of cell p ; chord of Γ_{qj} in R_p	m ⁻¹ , m
Θ_i	shelter density at station i , Eq. (16)	—
<i>Objective functionals</i>		
f_1	daily sensible-heat release (per km, see Section 4.3)	MJ d ⁻¹

Continued on the next page

Table 1 (continued)

Symbol	Meaning	Units / range
f_2	worst-case A-weighted level over the receiver set	dB(A)
f_3	roadside risk index (hazard less shelter benefit)	yr ⁻¹
f_4	annual irrigation volume delivered (per km, see Section 4.3)	m ³ yr ⁻¹

Table 2: Model parameters. All are data of the instance, and none is a decision variable.

Symbol	Meaning	Units
<i>Geometry and discretization</i>		
$L; \Delta, \delta$	corridor length; cell edges	m
w_m, w_c, w_v	median, carriageway, verge width	m
$y_i = w_m/2, y_o = y_i + w_c$	inner and outer edges of the travel way	m
$N = L/\Delta, n_c$	stations; cells	—
A_P, A_V	pavement and verge area	m ²
θ, ϕ	road azimuth (clockwise from north); site latitude	rad
<i>Species attributes, $s \in \mathcal{S}$</i>		
$h_s, r_s; \rho_s$	mature crown height and radius; foliage density	m, —
$d_s; \omega_s$	trunk diameter; water requirement at $\alpha = 1$	m, m ³ yr ⁻¹
<i>Radiation and surface energy partition</i>		
$I_b(t), E = \int_{\mathcal{T}} I_b dt$	beam irradiance; daily exposure	W m ⁻² , MJ m ⁻²
$\beta(t), \psi(t)$	solar altitude and azimuth	rad
$\gamma_{\text{pav}}, \gamma_{\text{shd}}, \gamma_{\text{soil}}$	partition of exposed and shaded pavement, bare soil	—
$\gamma_{\text{can}}^0, \gamma_{\text{can}}^1$	canopy partition, dry and transpiring	—
λ_v, ρ_w	latent heat; water density	J kg ⁻¹ , kg m ⁻³
<i>Acoustics</i>		
L'_W, ℓ_{seg}	source power per meter; segment length	dB(A) m ⁻¹ , m
$a_{\text{atm}}; \kappa, \bar{A}$	air absorption; foliage rate and cap	dB m ⁻¹ ; dB m ⁻¹ , dB
$\bar{A}_g; h_{\text{src}}, h_{\text{rec}}$	ground-effect cap; source, receiver height	dB; m

Continued on the next page

Table 2 (continued)

Symbol	Meaning	Units
<i>Roadside safety and aeolian shelter</i>		
$P_e; \lambda$	encroachment rate; lateral-extent decay	$\text{km}^{-1} \text{yr}^{-1}, \text{m}^{-1}$
v_d, a_r	departure speed; verge deceleration	$\text{m s}^{-1}, \text{m s}^{-2}$
c_b, d_{crit}, d_{ref}	frangible severity; threshold; reference diameter	—, m, m
W_{cz}, w_{defl}, y_b	clear zone; deflection envelope; barrier line	m
S, R_i, h_{sight}	sight distance; curve radius; height cap	m
h_{gl}^-, h_{gl}^+, G	glare-screen height window; maximum empty run	m, stations
$c_d, \nu, \zeta, \bar{h}$	dust rate per station; saturation; shelter decay; reference belt height	yr^{-1} , —, m, m
<i>Water</i>		
$\tau_z \geq 1; W, \alpha_{min}$	delivery multiplier; budget; survival sufficiency	—; $\text{m}^3 \text{yr}^{-1}$, —

3.2 Modeling Assumptions

The formulation rests on the following hypotheses, referred to as (A1)–(A11) wherever they are invoked.

(A1) Longitudinal homogeneity. Cross-sectional geometry, traffic composition and meteorology are invariant in x over $[0, L]$; horizontal curvature enters only through the sight-distance constraint (C5). Consequently every functional decomposes into a sum over the N stations.

(A2) Mature-state evaluation. Each planted individual is represented at its mature dimensions, h_s, r_s and d_s . Establishment, the growth trajectory, mortality and replacement are outside the model horizon, so f_1 – f_4 describe the asymptotic design state rather than a life-cycle average.

(A3) Cluster homogeneity. An activated cell carries n_s identical individuals of a single species, placed at the species' own intra-row spacing, so intra-cell crowns are pairwise disjoint by construction and no intra-cell overlap term is required.

- (A4) Zone-level irrigation.** Irrigation sufficiency is decided per zone, not per cell. This reflects the hydraulic reality that a verge or median is served by one delivery line, and it keeps the continuous part of the decision vector three-dimensional.
- (A5) Square-root allometric response.** Realised crown projected area responds linearly to irrigation sufficiency, whence $\tilde{r}_p \propto \sqrt{\alpha}$; the same exponent is applied to $\tilde{h}_p$ so that the crown aspect ratio h_s/r_s is preserved under water stress. The response is concave in α and saturates at the mature dimensions.
- (A6) Opaque spherical crowns over a horizontal plane.** Crowns are opaque spheres and the receiving surface is horizontal, so the shadow of one crown is an ellipse. Only the beam component is projected; diffuse and reflected radiation are not attributed to individual crowns.
- (A7) Additive shading with a physical cap.** Shadows cast by distinct clusters are summed and the total is truncated at the pavement area. Mutual overlap of shadows is neglected, which overstates A^{sh} when clusters are dense; the truncation in (8) bounds the resulting error.
- (A8) Clear-sky design day.** Radiation is evaluated on a single design day using a clear-sky beam model. Diffuse and longwave exchanges, cloud, and the seasonal cycle are excluded, and the partition coefficients $\gamma_\bullet$ act as reduced-form surrogates for a full surface energy balance. Because the omitted diffuse and longwave terms deposit additional energy in the same surfaces, f_1 is a lower-bound estimate of the true sensible-heat release of the cross-section, and the reductions of Section 9 are model-conditional rather than measurements; Section 8.5 quantifies their dependence on the $\gamma_\bullet$.
- (A9) Reduced ISO 9613-2 propagation.** The carriageways are incoherent line sources; contributions are energy-summed. Only geometrical divergence, atmospheric absorption, foliage attenuation and the ground effect are retained. Meteorological correction, barrier insertion loss and façade reflection are excluded, so f_2 is a downwind, screen-free estimate. All three omissions can only lower a receiver level, so f_2 is an upper-bound estimate and the attenuations reported below are conservative; Remark 2 shows that a decision-independent screen leaves every comparison between designs unchanged.
- (A10) Decision-independent ray geometry.** Source–receiver rays and hence the chords $\ell_{qj,p}$ depend on the cell partition and on the source and receiver positions alone, not on (ξ, α) ; see Remark 1.
- (A11) Expected-value safety accounting.** The encroachment–impact–severity chain and the shelter benefit are evaluated in expectation. Tail behavior, which dominates roadside safety, is not represented; a chance-constrained formulation [7] is the natural extension.

3.3 Geometry of the Cross-Section

Let the corridor occupy $\mathcal{C} = [0, L] \times \mathbb{R}$ with longitudinal coordinate x and transverse coordinate y measured from the highway centerline, and let θ be the road azimuth, i.e. the clockwise angle from north to the direction of increasing x . Write w_m for the median width, w_c for one carriageway width and w_v for the available verge depth, and set $y_i = w_m/2$, $y_o = y_i + w_c$. The cross-section (Figure 1) decomposes as

$$\begin{aligned}\mathcal{M} &= \{y : |y| \leq y_i\}, & \mathcal{P} &= \{y : y_i \leq |y| \leq y_o\}, \\ \mathcal{V}^\pm &= \{y : \pm y \in [y_o, y_o + w_v]\},\end{aligned}$$

the median, the paved travel way, and the two verges, with corresponding areas $A_p = 2w_cL$ and $A_v = 2w_vL$. Verges and slopes bordering highways are in practice managed as small, discretized planting units whose species composition differs systematically between them [20], which is precisely the structure the cell partition below is designed to represent. We write $\sigma \in \{+, -\}$ for a side, σ_R for the side carrying the residential receivers and σ_W for the side facing the prevailing wind. These two labels are data of the instance; they are what breaks the symmetry between the verges.

The planting domain $\mathcal{M} \cup \mathcal{V}^+ \cup \mathcal{V}^-$ is partitioned into rectangular cells of dimensions $\Delta \times \delta$. Cell p has center (x_p, y_p) , footprint $R_p = [x_p \pm \Delta/2] \times [y_p \pm \delta/2]$, zone label $z(p) \in \{0, -, +\}$ and station index $\iota(p) \in \{1, \dots, N\}$ with $N = L/\Delta$; a station is the set of all cells sharing a longitudinal slice of length Δ . The cell set is $\mathcal{Q} = \mathcal{Q}_0 \cup \mathcal{Q}_- \cup \mathcal{Q}_+$ with $|\mathcal{Q}| = n_c$. Finally, the lateral offset

$$y_p^{off} = \begin{cases} y_i - |y_p|, & p \in \mathcal{Q}_0, \\ |y_p| - y_o, & p \in \mathcal{Q}_\pm, \end{cases} \quad (1)$$

measures the distance from cell p to the nearest travel-lane edge and is the argument of every safety relation in Section 4.

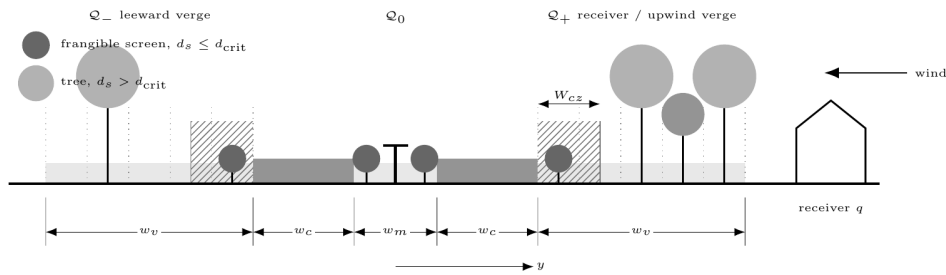


Figure 1: Cross-sectional domain. The median $\mathcal{Q}_0$ lies behind a barrier and is governed by a bilateral clear zone, so it admits frangible screening material only; the verges $\mathcal{Q}_\pm$ are governed by a unilateral clear zone (hatched). Cell columns are drawn at the five transverse offsets of the numerical instance. The receiver set and the prevailing wind both lie on the $+$ side.

3.4 Species Catalogue and Cluster Cardinality

Each species s in the finite catalogue $\mathcal{S}$ carries the attribute tuple $(h_s, r_s, \rho_s, d_s, \omega_s)$ defined in Table 2. Activating a cell with species s installs a cluster of

$$n_s = \max\{1, \lfloor \Delta/2r_s \rfloor\} \quad (2)$$

individuals: the floor $\lfloor \Delta/2r_s \rfloor$ counts the crowns of diameter $2r_s$ that fit along the longitudinal edge of the cell without touching, and the surrounding $\max\{1, \cdot\}$ operator guarantees that a cell can host at least one individual even when $2r_s > \Delta$, the resulting longitudinal overhang being controlled by the spacing constraint (C6). Two points deserve emphasis. First, n_s is evaluated at the *mature* radius r_s and not at the realized radius $\tilde{r}_p$, so that cluster cardinality is a fixed integer parameter of the species and does not become a decision-dependent integer, which would introduce a second layer of discreteness. Second, this device ties the combinatorial dimension of the program to the cell partition, $|\mathcal{Q}| \times |\mathcal{S}|$, rather than to the individual plant.

3.5 Decision Variables and the Irrigation–Allometry Coupling

The decision variables are the binary activation array and one continuous irrigation-sufficiency variable per zone,

$$\begin{aligned} \xi_{p,s} &\in \{0, 1\}, & p \in \mathcal{Q}, s \in \mathcal{S}, \\ \alpha_z &\in [\alpha_{\min}, 1], & z \in \{0, -, +\}. \end{aligned} \quad (3)$$

Here $\xi_{p,s} = 1$ states that cell p is planted with species s , and α_z is the fraction of the unstressed water requirement actually delivered to zone z , bounded below by the survival threshold $\alpha_{\min} > 0$. We write $u_p = \sum_{s \in \mathcal{S}} \xi_{p,s} \in \{0, 1\}$ for the occupancy indicator and $s(p)$ for the species assigned to an occupied cell, so that $u_p = 1 \Leftrightarrow \xi_{p,s(p)} = 1$.

By (A5), realized crown geometry responds to irrigation sufficiency through

$$\tilde{r}_p = r_{s(p)} \sqrt{\alpha_{z(p)}}, \quad \tilde{h}_p = h_{s(p)} \sqrt{\alpha_{z(p)}}. \quad (4)$$

The exponent follows from the assumption that projected crown area, and with it transpiring leaf area, scales linearly with the water actually supplied: $\pi \tilde{r}^2 = \alpha \pi r_s^2$ gives $\tilde{r} = r_s \sqrt{\alpha}$, and applying the same factor to $\tilde{h}$ leaves h_s/r_s invariant under stress. At $\alpha = 1$ the mature dimensions are recovered. Because $\tilde{r}_p$ and $\tilde{h}_p$ carry the product $\xi_{p,s} \sqrt{\alpha_z}$, every functional in which they appear is bilinear in the mixed variables, which is the source of the nonconvexity of (P) .

3.6 The Multi-Objective Program

The feasible set $\mathcal{F} \subset \{0, 1\}^{|\mathcal{Q}| \times |\mathcal{S}|} \times [\alpha_{\min}, 1]^3$ collects the constraints (C1)–(C9) developed in Section 5. The program is

$$(P) \quad \begin{aligned} & \min_{(\xi, \alpha) \in \mathcal{F}} (f_1(\xi, \alpha), f_2(\xi, \alpha), f_3(\xi, \alpha)) \\ & \text{s.t.} \quad f_4(\xi, \alpha) \leq W, \end{aligned} \quad (5)$$

with f_4 the annual irrigation volume and W the allocated budget, treated as an ε -constraint so that the reported fronts are indexed by W . Efficiency is understood in the usual Pareto sense: $(\xi, \alpha) \in \mathcal{F}$ is efficient if no feasible point (ξ', α') satisfies $f_k(\xi', \alpha') \leq f_k(\xi, \alpha)$ for all $k \in \{1, 2, 3\}$ with at least one strict inequality [14, 26]. Since ξ is binary and $\mathcal{F}$ compact in α , the efficient set is nonempty whenever $\mathcal{F} \neq \emptyset$.

4 Objective Functionals

The four functionals are constructed in the order described in Section 4.1. Section 4.2 derives the shadow projection on which the thermal load of Section 4.3 rests, and Sections 4.4, 4.5, and 4.6 define the receiver-noise, roadside-risk, and irrigation-demand functionals, respectively.

4.1 Order of Construction

The four functionals are built from (ξ, α) in a fixed sequence, each step using only quantities established in the preceding ones: $(\xi, \alpha) \mapsto (u_p, s(p))$ by (3); $(u_p, s(p), \alpha_{z(p)}) \mapsto (\tilde{r}_p, \tilde{h}_p)$ by (4); then $(\tilde{r}_p, \tilde{h}_p) \mapsto (A^{sh}(t), A^c)$ by Lemma 1 and (8), $\mapsto \varepsilon_p$ by (12), $\mapsto \Theta_i$ by (16), $\mapsto m_w$ by (9); and finally these aggregates enter f_1, f_2, f_3, f_4 through (10), (14), (17) and (18). Only the third step requires geometry beyond the cell partition, and within it only the chord lengths $\ell_{qj,p}$ are expensive; by (A10) these are computed once and cached. The remaining evaluations are closed-form sums over $\mathcal{Q}$.

4.2 Shadow Projection and the Role of the Road Azimuth

Let $\beta(t)$ and $\psi(t)$ denote solar altitude and solar azimuth measured clockwise from north on the design day, both obtained from the site latitude ϕ and the date by standard position formulae [13].

Lemma 1 (Shadow displacement). Under (A6), a spherical crown of radius r whose center is at height h above the horizontal plane and at transverse coordinate y projects a shadow whose center lies at transverse coordinate

$$y^{sh}(t) = y - h \cot \beta(t) \sin(\psi(t) - \theta), \quad (6)$$

and whose transverse half-extent is

$$a_{\perp}(t) = r \sqrt{\sin^{-2} \beta(t) \sin^2(\psi(t) - \theta) + \cos^2(\psi(t) - \theta)}. \quad (7)$$

Proof. In east–north coordinates the road axis and its normal are

$$e_{\parallel} = (\sin \theta, \cos \theta), \quad e_{\perp} = (\cos \theta, -\sin \theta).$$

The horizontal shadow displacement of a point at height h is $-h \cot \beta (\sin \psi, \cos \psi)$, whose component along $e_{\perp}$ is

$$-h \cot \beta (\sin \psi \cos \theta - \cos \psi \sin \theta) = -h \cot \beta \sin(\psi - \theta),$$

which gives (6). The shadow of a sphere of radius r on the horizontal plane is an ellipse with semi-axis $r / \sin \beta$ along the displacement direction and r across it; the displacement direction has component $\sin(\psi - \theta)$ on $e_{\perp}$, and (7) follows by composing the two semi-axes. $\square$

Corollary 1 (Azimuthal degeneracy). If $\psi(t) \equiv \theta \pmod{\pi}$ then $y^{sh}(t) = y$ and $a_{\perp}(t) = r$: the canopy transfers no shade laterally. The transfer is maximal when $\psi - \theta = \pm\pi/2$. Consequently the shading component of the thermal functional depends on the road azimuth through $\sin(\psi - \theta)$, and its sign—hence the side of the corridor that benefits—reverses at solar noon.

Corollary 1 is the first source of transverse asymmetry: for a corridor of fixed azimuth, morning and afternoon shading are delivered by opposite verges, and at latitudes where β approaches $\pi/2$ near noon the displacement $h \cot \beta$ collapses, so that verge canopy standing beyond a clear zone of width W_{cz} cannot reach the pavement at all during the hours of peak irradiance.

4.3 Corridor Thermal Load

Let $I_b(t)$ be beam irradiance on the horizontal, obtained from a clear-sky model [13], and $E = \int_{\mathcal{T}} I_b dt$ the daily beam exposure. The shaded pavement area is assembled in three steps. (i) By (A6) the shadow of a single crown of realized radius $\tilde{r}_p$ is an ellipse of semi-axes $\tilde{r}_p / \sin \beta(t)$ and $\tilde{r}_p$, hence of area $\pi \tilde{r}_p^2 / \sin \beta(t)$; a cluster of $n_{s(p)}$ disjoint crowns casts $n_{s(p)}$ such ellipses.

(ii) The shadow of cell p occupies the transverse interval $J_p(t) = [y_p^{sh}(t) - a_{\perp,p}(t), y_p^{sh}(t) + a_{\perp,p}(t)]$, with y_p^{sh} and $a_{\perp,p}$ given by (6)–(7) evaluated at $(\tilde{r}_p, \tilde{h}_p, y_p)$; the useful fraction is $\mu_p(t) = |J_p(t) \cap \mathcal{P}| / 2a_{\perp,p}(t) \in [0, 1]$. (iii) Summing over cells and truncating at the pavement area, as licensed by (A7),

$$\begin{aligned} A^{sh}(t) &= \min \left\{ A_{\mathcal{P}}, \sum_{p \in \mathcal{Q}} u_p n_{s(p)} \frac{\pi \tilde{r}_p^2}{\sin \beta(t)} \mu_p(t) \right\}, \\ A^c &= \min \left\{ A_{\mathcal{V}}, \sum_{p \in \mathcal{Q}_{\pm}} u_p n_{s(p)} \pi \tilde{r}_p^2 \right\}, \end{aligned} \quad (8)$$

the verge canopy cover A^c being obtained in the same way but without projection, since it is the plan area of the crowns themselves.

Each surface releases a fraction γ of its absorbed beam energy as sensible heat to the corridor air layer, the remainder being stored or partitioned to latent flux; these coefficients are reduced-form surrogates in the sense of (A8). We take γ_{pav} for exposed pavement, $\gamma_{\text{shd}} < \gamma_{\text{pav}}$ for shaded pavement, γ_{soil} for bare verge soil, and for canopy a value interpolating between a non-transpiring and a fully transpiring state,

$$\begin{aligned} \gamma_{\text{can}}(\Lambda) &= \gamma_{\text{can}}^0 - (\gamma_{\text{can}}^0 - \gamma_{\text{can}}^1) \Lambda, \\ \Lambda &= \min \left\{ 1, \frac{\lambda_v m_w}{E A^c} \right\}, \\ m_w &= \frac{\rho_w}{365} \sum_{p \in \mathcal{Q}} u_p n_{s(p)} \omega_{s(p)} \alpha_{z(p)}. \end{aligned} \quad (9)$$

Here m_w is the mean daily delivered mass, so $\lambda_v m_w$ is the latent-heat flux the delivered water could sustain over one day and $E A^c$ the beam energy the canopy intercepts over the same day. Their ratio $\Lambda \in [0, 1]$, capped at unity, states that transpiration is the smaller of the energy-limited and the water-limited rate. Collecting the four surface contributions, the thermal functional is

$$\begin{aligned} f_1 &= \gamma_{\text{pav}} \int_{\mathcal{T}} I_b (A_{\mathcal{P}} - A^{sh}) dt + \gamma_{\text{shd}} \int_{\mathcal{T}} I_b A^{sh} dt \\ &\quad + \gamma_{\text{soil}} E (A_{\mathcal{V}} - A^c) + \gamma_{\text{can}}(\Lambda) E A^c, \end{aligned} \quad (10)$$

whose four terms are, in order, exposed pavement, shaded pavement, bare verge soil and canopy. The first two are time-resolved because A^{sh} moves with the sun; the last two are not, since canopy cover is static. The units are those of EA , namely MJ d^{-1} for the modeled corridor of length L . Because every area in (10) is proportional to L under (A1), f_1/L is the corridor-length-normalized load; we report it throughout in $\text{GJ km}^{-1} \text{d}^{-1}$, which for the instance of Section 8 ($L = 1 \text{ km}$) is numerically f_1 in GJ d^{-1} . The same normalization is applied to f_4 , reported in $\text{m}^3 \text{yr}^{-1} \text{km}^{-1}$.

4.4 Receiver Noise

Traffic on each carriageway is modeled, per (A9), as an incoherent line source discretized into segments $j \in \mathcal{J}$ of length ℓ_{seg} , each carrying the A-weighted power level $L_W = L'_W + 10 \log_{10} \ell_{seg}$, where L'_W is the source power per unit length; $\mathcal{R}$ is the receiver set, positioned at the façades of the adjacent housing on side σ_R . Following the reduced form of ISO 9613-2 [19], the level at receiver q from segment j is

$$L_{qj} = L_W - \underbrace{10 \log_{10}(2\pi d_{qj}^2)}_{\text{divergence}} - \underbrace{a_{atm} d_{qj}}_{\text{air absorption}} - \underbrace{A_{qj}^{fol}}_{\text{foliage}} - \underbrace{A^{gr}}_{\text{ground}}, \quad (11)$$

with d_{qj} the source–receiver distance and a_{atm} the atmospheric absorption coefficient. Foliage attenuation accumulates along the ray Γ_{qj} joining source to receiver,

$$\begin{aligned} A_{qj}^{fol} &= \min\left\{\bar{A}, \kappa \sum_{p \in \mathcal{Q}} \varepsilon_p \ell_{qj,p}\right\}, \\ \varepsilon_p &= u_p \rho_{s(p)} \min\left\{1, \frac{n_{s(p)} \pi \tilde{r}_p^2}{\Delta \delta}\right\}, \\ \ell_{qj,p} &= |\Gamma_{qj} \cap R_p|. \end{aligned} \quad (12)$$

The cell extinction ε_p is the species foliage density $\rho_{s(p)}$ scaled by the plan-area cover ratio of the cluster within its own cell, capped at unity so that a cell cannot be more than fully occupied; κ converts extinction-weighted path length into decibels, and $\bar{A}$ caps the term as required by the standard.

Remark 1. The chord lengths $\ell_{qj,p}$ depend only on the cell partition and the source–receiver geometry, not on the decision variables (A10). They are computed once by a slab intersection and stored; A^{fol} is then linear in the cell extinctions ε_p up to the cap $\bar{A}$. This is what makes the acoustic functional cheap to evaluate inside a population-based search.

Ground attenuation is taken proportional to the vegetated fraction of the receiver-side verge,

$$A^{gr} = \bar{A}_g \frac{|\{p \in \mathcal{Q}_{\sigma_R} : u_p = 1\}|}{|\mathcal{Q}_{\sigma_R}|}, \quad (13)$$

reflecting the replacement of an acoustically hard surface by a porous litter layer, which is the dominant mechanism by which vegetation belts attenuate traffic noise [24, 31]; $\bar{A}_g$ is the attenuation of a fully vegetated verge. The functional is the worst-case receiver level, obtained by energy summation over segments and maximization over receivers,

$$f_2 = \max_{q \in \mathcal{R}} 10 \log_{10} \sum_{j \in \mathcal{J}} 10^{L_{qj}/10}, \quad (14)$$

which is non-smooth by construction, both through the maximum and through the two caps in (12); this is one of the two reasons—the other being Proposition 3—why Section 7 uses a population heuristic rather than an exact mixed-integer method.

Remark 2 (Barrier insertion loss). Let $D \geq 0$ be the insertion loss of a screen that is present in the instance but is not a decision variable, so that D applies to every source–receiver path alike. Replacing L_{qj} by $L_{qj} - D$ in (11) multiplies every term of the energy sum in (14) by $10^{-D/10}$, and neither cap in (12) nor the ground term (13) depends on D ; hence $f_2 \mapsto f_2 - D$ exactly. A fixed screen thus translates the f_2 axis and changes no comparison between designs: the efficient set of (P) , the ranking of designs and every attenuation *difference* reported in Section 8 are invariant, while the absolute levels are upper bounds. In the instance of Section 8 the median barrier is a high-tension cable system—posts and wire ropes, with no continuous solid surface—for which D is negligible in any case, and constraint (C4) credits only its mechanical deflection envelope, not any acoustic screening. There is therefore no inconsistency between excluding barrier insertion loss from f_2 and retaining the barrier in the admissibility rules of Section 5. The do-nothing baseline quoted in Section 8.1 contains no barrier screening.

4.5 Roadside Risk

Recall y_p^{off} from (1). The expected fixed-object consequence of a single individual of species s standing at offset y follows the encroachment–impact–severity chain used in roadside safety analysis [3]:

$$\eta(y, s) = \underbrace{P_e}_{\text{encroach}} \underbrace{e^{-\lambda y}}_{\text{reach}} \underbrace{c_s}_{\text{severity}} \underbrace{\left(1 - \frac{2a_r y}{v_d^2}\right)_+}_{\text{residual energy}}, \quad (15)$$

$$c_s = \begin{cases} c_b, & d_s \leq d_{crit}, \\ \min\{1, (d_s/d_{ref})^2\}, & \text{otherwise.} \end{cases}$$

where P_e is the encroachment rate, $e^{-\lambda y}$ the probability that the lateral extent of an encroachment exceeds y , v_d the departure speed and a_r the deceleration available on the verge, so that $(1 - 2a_r y/v_d^2)_+$ is the fraction of the departure kinetic energy still carried at offset y and vanishes beyond the run-out distance $v_d^2/2a_r$. The severity coefficient c_s collapses to $c_b \ll 1$ for stems below the frangibility threshold d_{crit} and otherwise grows with the square of the trunk diameter, normalized by d_{ref} ; this is the mechanism by which species selection, and not merely siting, enters the risk objective.

Against this stands the shelter benefit. A vegetation belt on the upwind side reduces crash exposure during dust events [25] by raising visibility and damping lateral gusts, with an effect that saturates in belt optical density and decays with distance from the protected surface over a scale ζ proportional to belt height [1, 10]. Per (A1) the benefit is accumulated station by station: writing $\bar{h}$ for a reference belt height that renders the ratio dimensionless,

$$\Theta_i = \sum_{p \in \mathcal{Q}_{\sigma_W}: \iota(p)=i} u_p \frac{\rho_{s(p)} \bar{h}_p}{\bar{h}} e^{-y_p^{off}/\zeta}, \quad (16)$$

so that Θ_i is an optical-density index of the upwind belt at station i , in which taller and denser material counts more and remote material is discounted exponentially. The realized benefit saturates in Θ_i at rate ν , and c_d is the dust-attributable expected rate of severe outcomes per station and per year that a fully developed belt removes. The risk functional is therefore

$$\begin{aligned} f_3 = & \underbrace{\sum_{p \in \mathcal{Q}} u_p n_{s(p)} \eta(y_p^{off}, s(p))}_{\text{fixed-object hazard}} \\ & - \underbrace{c_d \sum_{i=1}^N (1 - e^{-\nu \Theta_i})}_{\text{aeolian shelter benefit}}, \end{aligned} \quad (17)$$

negative values denoting a net safety gain. Both terms are expected rates of severe outcomes per year over the modeled corridor, so f_3 has units yr^{-1} and the two are directly comparable; because P_e is quoted per kilometer and per year, this reading is exact for the instance of Section 8, where $L = 1$ km, and for a corridor of different length P_e must be rescaled to the station length Δ before (15) is applied.

4.6 Irrigation Demand

In arid urban settings the water needed to sustain planted surfaces is set jointly by species water use and by the surface energy exchanges of the built cross-section, so irrigation volume behaves as a binding design resource rather than a fixed overhead [30]. Accordingly we take

$$f_4 = \sum_{p \in \mathcal{Q}} u_p n_{s(p)} \omega_{s(p)} \alpha_{z(p)} \tau_{z(p)}, \quad (18)$$

the product, cell by cell, of cluster cardinality, unstressed species requirement ω_s , delivered fraction α_z , and a delivery and maintenance multiplier $\tau_z \geq 1$. The last accounts for conveyance losses and access cost, with $\tau_0 > \tau_{\pm}$ because median irrigation requires carriageway crossing. Note that f_4 is linear in α_z whereas f_1 is concave in it through (4): the design trade-off between

deficit irrigation of many cells and full irrigation of few is a direct consequence of this mismatch of curvature.

5 Constraints

The feasible set $\mathcal{F}$ is defined by the following, for all $p \in \mathcal{Q}$, $s \in \mathcal{S}$, $i \in \{1, \dots, N\}$. Constraints (C1) and (C9) delimit the decision space; (C2)–(C4) express roadside-safety admissibility and act as a fixed mask on the pairs (p, s) ; (C5)–(C8) are operational requirements that couple the binaries to the continuous variables through the realized dimensions $\tilde{r}_p, \tilde{h}_p$ of (4), and are therefore the constraints that make the feasible set depend on α .

(C1) Assignment. $\sum_{s \in \mathcal{S}} \xi_{p,s} \leq 1$: a cell hosts at most one species, so u_p and $s(p)$ are well defined.

(C2) Verge clear zone. For $p \in \mathcal{Q}_{\pm}$ with $y_p^{off} < W_{cz}(v_d, ADT, \text{slope})$ and $d_s > d_{crit}$: $\xi_{p,s} = 0$. Only frangible material is admitted inside the recoverable width [4]. Here W_{cz} is the clear-zone width, a tabulated function of design speed, average daily traffic ADT and side slope, and d_{crit} the frangibility threshold of (15).

(C3) Median bilateral clear zone. For $p \in \mathcal{Q}_0$ the recoverable width is measured from both travel-way edges, so $\xi_{p,s} = 0$ whenever $d_s > d_{crit}$ and $w_m/2 < W_{cz}$. A median narrower than twice the clear zone therefore admits frangible screening material only, irrespective of its nominal width.

(C4) Barrier deflection. Where a median barrier standing on the line $y = y_b$ is present, (C3) is relaxed for cells satisfying $|y_p - y_b| \geq w_{defl}$, the dynamic deflection envelope of the barrier system, i.e. the lateral excursion of the barrier under design impact, within which no rigid object may stand. A planting column lying on the barrier line itself is never relaxed by (C4).

(C5) Sight distance. On a horizontal curve of radius R_i with required stopping sight distance S , the middle ordinate is $M_i = R_i(1 - \cos(S/2R_i))$, the lateral clearance that must be kept free of visual obstruction. For cells on the inside of the curve with $|y_p - y_i^{in}| < M_i$, where y_i^{in} is the transverse coordinate of the inner travel-lane edge at station i , the height is capped: $\tilde{h}_p \leq h_{sight}$. On a tangent alignment $M_i \equiv 0$ and (C5) is vacuous.

(C6) Crown spacing. For p, p' in the same zone,

$$\xi_{p,s} + \xi_{p',s'} \leq 1 \quad \text{whenever} \quad \|(x_p, y_p) - (x_{p'}, y_{p'})\| < \tilde{r}_p + \tilde{r}_{p'},$$

so that crowns of distinct cells may not interpenetrate. The threshold depends on α , so the conflict graph is itself a function of the continuous variables.

(C7) Glare-screen continuity. On median segments requiring headlight screening, no run of more than G consecutive stations may be empty,

$$\sum_{i'=i}^{i+G-1} \sum_{p \in \mathcal{Q}_0: \iota(p)=i'} \sum_{s \in \mathcal{S}_{scr}} \xi_{p,s} \geq 1, \quad i \leq N - G + 1.$$

The binding form is a *maximum gap*, not a minimum spacing: periodic interruption of a screen produces a stroboscopic luminance flicker that is itself a hazard.

(C8) Screen height window. $h_{gl}^- \leq \tilde{h}_p \leq h_{gl}^+$ for $p \in \mathcal{Q}_0$, $s \in \mathcal{S}_{scr}$: high enough to intercept the opposing headlight beam, low enough to leave the driver's sight line clear. Since $\tilde{h}_p$ scales with $\sqrt{\alpha_0}$, the lower bound places a floor on median irrigation, which is the mechanism behind Proposition 4.

(C9) Budget and domain. $f_4 \leq W$; $\xi_{p,s} \in \{0, 1\}$; $\alpha_z \in [\alpha_{\min}, 1]$.

Constraints (C1)–(C8) are enforced exactly by the repair operator of Section 7, while (C9) is handled by constraint domination; the rationale for that split is given there.

6 Structural Properties

This section establishes three structural properties of (P) : the non-monotone lateral risk response (Section 6.1), the incompatibility of side selection (Section 6.2), and the computational complexity of the scalarized problem (Section 6.3).

6.1 Non-Monotone Lateral Risk Response

Consider the marginal effect of activating a single cell at lateral offset y with species s , evaluated at $\Theta \rightarrow 0^+$ so that the saturating shelter term is linearized. Writing $A_s = n_s P_e c_s$, $b = 2a_r/v_d^2$ and $C_s = c_d \nu \rho_s h_s / \bar{h}$, this marginal response is

$$\varphi_s(y) = A_s e^{-\lambda y} (1 - by)_+ - C_s e^{-y/\zeta}. \quad (19)$$

Proposition 1 (Interior minimizer). Suppose $\lambda > \zeta^{-1}$ and

$$A_s \zeta (\lambda + b) > C_s. \quad (20)$$

Then φ_s is strictly decreasing on $[0, y_s^*]$ and strictly increasing on (y_s^*, ∞) with $\varphi_s(y) \uparrow 0^-$ as $y \rightarrow \infty$; the minimizer $y_s^* \in (0, b^{-1})$ is the unique root of

$$A_s[\lambda(1 - by) + b]e^{-\lambda y} = \zeta^{-1}C_s e^{-y/\zeta}. \quad (21)$$

Moreover φ_s changes sign exactly once, at $y_s^0 < y_s^*$, if and only if $A_s > C_s$.

Proof. On $[0, b^{-1})$ differentiation of (19) gives $\varphi'_s(y) = -A_s[\lambda(1 - by) + b]e^{-\lambda y} + \zeta^{-1}C_s e^{-y/\zeta}$, so $\varphi'_s(y) = 0$ is equivalent to $g(y) = 1$ with $g(y) = A_s\zeta C_s^{-1}[\lambda(1 - by) + b]e^{-(\lambda - \zeta^{-1})y}$. Since $\lambda > \zeta^{-1}$, both factors of g are positive and strictly decreasing on $[0, b^{-1})$, so g is strictly decreasing there; condition (20) is exactly $g(0) > 1$, and $g(b^{-1}) = A_s\zeta b C_s^{-1}e^{-(\lambda - \zeta^{-1})/b} < 1$ for the parameter ranges of interest, so g crosses unity once. Hence $\varphi'_s < 0$ on $[0, y_s^*)$ and $\varphi'_s > 0$ on (y_s^*, b^{-1}) . For $y \geq b^{-1}$ the hazard term vanishes identically and $\varphi_s(y) = -C_s e^{-y/\zeta}$ is strictly increasing with limit 0^- . Continuity at b^{-1} completes the monotonicity claim. For the sign, $\varphi_s(0) = A_s - C_s$ and $\varphi_s < 0$ on a neighborhood of $+\infty$; strict unimodality then permits exactly one crossing, which exists if and only if $\varphi_s(0) > 0$. $\square$

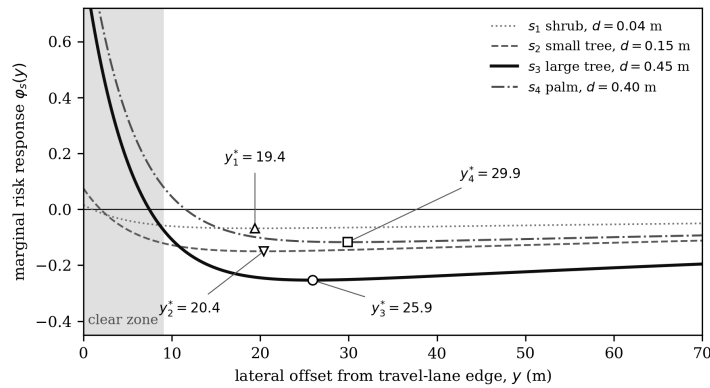


Figure 2: Marginal risk response φ_s of (19) for the four species of the numerical instance. Condition (20) holds for all four, so each curve has a unique interior minimizer y_s^* (markers); for s_1 and s_2 the minimum is shallow but present. Species s_3 and s_4 are net liabilities near the travel way and net benefits beyond the sign change. The palm s_4 still satisfies $\varphi_4(W_{cz}) > 0$.

Remark 3 (Verification of (20) on the catalogue). Both hypotheses must be checked on the data rather than assumed. For the catalogue of Table 4, $\lambda = 0.18 \text{ m}^{-1}$ against $\zeta^{-1} = 0.0067 \text{ m}^{-1}$, and condition (20) is satisfied by every species by more than an order of magnitude, using the A_s, C_s listed there. The proposition therefore applies to the whole catalogue: each species has a unique interior minimizer, the four are ordered $y_1^* < y_2^* < y_3^* < y_4^*$, and all lie strictly beyond $W_{cz} = 9 \text{ m}$. What distinguishes s_1 and s_2 is only the *depth* of the minimum, an order of magnitude smaller, which is why their minima are barely visible in Figure 2. The sign-change offsets differ far more sharply: y_s^0 runs from 1.0 m for the frangible shrub, a net benefit

essentially everywhere it may stand, to 11.5 m for the palm, still a net liability at the clear-zone boundary.

Remark 4. Proposition 1 formalizes a fact that setback-based guidance cannot express: the safety-optimal offset is an interior point, not the boundary of the clear zone. For the instance of Section 8 the large-trunk species has $y_3^0 = 7.4$ m and $y_3^* = 25.9$ m against a clear zone of $W_{cz} = 9$ m, so the offset minimizing net risk is nearly three times the recoverable width (Figure 2). The sparse-crowned palm is the least favourable species in this catalogue under the risk functional (17): a rigid stem of comparable diameter combined with low foliage density gives $\varphi_4(W_{cz}) > 0$, i.e. it remains a net liability at the clear-zone boundary. It is not least favourable under f_1 or f_4 , as Section 8.3 shows. Because φ_s is unimodal but not convex on $[0, \infty)$, and because φ_s is not monotone, greedy outward displacement is not an optimal policy and exchange arguments on the offset variable fail.

6.2 Side-Selection Incompatibility

Proposition 2 (Asymmetry of efficient designs). Let $\sigma_R = \sigma_W = \sigma$ and let Γ_{qj} denote the source–receiver rays. Then:

- (i) $\Gamma_{qj} \cap R_p = \emptyset$ for every $p \in \mathcal{Q}_{-\sigma}$, hence f_2 does not depend on $\xi|_{\mathcal{Q}_{-\sigma}}$;
- (ii) transferring an activated cluster from $p \in \mathcal{Q}_{-\sigma}$ to a free cell $p' \in \mathcal{Q}_{\sigma}$ with $y_{p'}^{off} = y_p^{off}$ and the same species leaves f_4 and the cover term of f_1 unchanged, does not increase f_2 , and strictly decreases f_3 by $c_d(e^{-\nu\Theta_i} - e^{-\nu(\Theta_i + \vartheta_{p'})}) > 0$, where $i = \iota(p')$ and $\vartheta_{p'} = \rho_{s(p')} \tilde{h}_{p'} e^{-y_{p'}^{off}/\zeta} / \bar{h}$ is the shelter increment contributed by p' in (16);
- (iii) consequently, if $A^c \leq A_V/2$ and the shading term of f_1 is neglected, every efficient design satisfies $\xi|_{\mathcal{Q}_{-\sigma}} \equiv 0$; if $A^c > A_V/2$ no such design is admissible and f_3 exceeds its one-sided optimum by a strictly positive amount increasing in A^c .

Proof. (1) Sources lie on $|y| \leq y_i + w_c/2$ and receivers on the σ side beyond the verge; every ray is a segment joining them, so its transverse coordinate stays within the interval spanned by its endpoints, which excludes $\mathcal{Q}_{-\sigma} \subset \{\text{sign}(y) = -\sigma, |y| \geq y_o\}$. Hence $\ell_{qj,p} = 0$ in (12). (2) The transfer preserves species, cardinality and offset, so the hazard sum in (17) and the demand (18) are unchanged; A^c is unchanged, so the third and fourth terms of (10) are unchanged; f_2 can only fall, since a positive $\varepsilon_{p'}$ is added to a nonnegative sum in (12). The shelter sum gains the strictly positive increment stated, because $1 - e^{-\nu\Theta}$ is strictly increasing. (3) Apply (2) repeatedly while free cells remain on $\mathcal{Q}_{\sigma}$; capacity is exhausted exactly when $A^c = A_V/2$. $\square$

Proposition 2 states that the transverse asymmetry of the optimum is structural rather than incidental: it follows from the fact that f_2 is blind to one verge while f_3 is not, and it holds for any parameterization satisfying the sign conditions.

Remark 5 (Scope of part (3)). Part (3) is proved with the shading term of f_1 suppressed, and that hypothesis is *not* satisfied by the model actually solved in Section 8, where the term is present. The gap is not cosmetic. By Corollary 1 the lateral shadow displacement $-h \cot \beta \sin(\psi - \theta)$ reverses sign at solar noon, so of the two verges exactly one can throw shade onto the travel way in the morning and the other in the afternoon; which one dominates is fixed by θ together with the hours at which $\cot \beta$ is large. For the instance azimuth $\theta = 75^\circ$ that contribution comes from the leeward verge, and from it alone: a single column of s_3 at the outermost offset delivers 4.9 GJ d^{-1} of pavement shading from the leeward verge and exactly zero from the receiver verge, the receiver-side shadow being displaced away from the travel way at every hour of the design day. Transferring a cluster as in (2) therefore leaves f_4 and the cover term unchanged, weakly improves f_2 and strictly improves f_3 but *may worsen the shading term of f_1* , so it need not be a Pareto improvement, and the conclusion $\xi|_{\mathcal{Q}-\sigma} \equiv 0$ does not survive.

What survives is the direction of the effect: efficient designs are biased toward the receiver side, not supported on it. Section 8.4 should be read accordingly—the receiver-side share it reports is high but strictly below one at every budget, and it *falls* as the budget grows, which is exactly what a competing shading term that recruits leeward cells predicts. Suppressing the shading term is thus the price of a clean statement, and we prefer to state the price than to claim the numerics confirm (3) literally.

6.3 Complexity

Proposition 3. The ε -constraint scalarization of (P) is NP-hard, and strongly NP-hard when (C6) is active.

Proof. Restrict to a single verge, one species, α fixed, $\delta > 2r_s$ so that (C6) is vacuous, no screening requirement, and ε loose on f_2, f_3 . Minimising (10) then reduces to maximizing $\sum_p v_p u_p$ with $v_p = (\gamma_{\text{soil}} - \gamma_{\text{can}}) E n_s \pi \tilde{r}^2$ subject to $\sum_p w_p u_p \leq W$, $w_p = n_s \omega_s \tau$, which is 0–1 Knapsack [17]. When (C6) is active the conflict graph induced by $\|\cdot\| < \tilde{r}_p + \tilde{r}_{p'}$ on cell centers is a unit-disk graph, and maximum independent set on unit-disk graphs is strongly NP-hard [9]. $\square$

Together with the non-smoothness of (14), the caps in (10)–(12) and the bilinear terms $\xi_{p,s} \alpha_z$ introduced by (4), Proposition 3 rules out exact mixed-integer linear treatment at realistic corridor lengths and motivates the population-based approach of Section 7.

Proposition 4 (Feasibility floor). $\mathcal{F} \neq \emptyset$ if and only if the catalogue contains a screening species $s^\dagger \in \mathcal{S}_{scr}$ with $d_{s^\dagger} \leq d_{crit}$, $2r_{s^\dagger} \leq \delta$ and $h_{gl}^- \leq h_{s^\dagger} \sqrt{\alpha_{\min}}$. In that case (C7) forces at least $\lceil N/G \rceil$ median activations and the irrigation budget must satisfy $W \geq w_{\min} = \lceil N/G \rceil n_{s^\dagger} \omega_{s^\dagger} \alpha_{\min} \tau_0$.

7 Solution Method

Exact global solution of a nonconvex mixed-integer program of this size is impractical at the cell counts of interest, the more so because the shading terms of Section 4 introduce trigonometric nonlinearities, for which deterministic global algorithms remain restricted to instances far smaller than ours [27]. We therefore approximate the Pareto front of (P) with NSGA-II [11], using constraint-domination for the budget and a repair operator for the combinatorial constraints.

Two scalarization choices deserve a word. Irrigation volume enters as an ε -constraint rather than as a fourth weighted term because f_4 is an allocated budget and not a preference: a weighted sum would trade water against the other three objectives at an exchange rate no allocating authority has set, whereas the ε -constraint returns one front per budget and leaves that trade-off explicit. The same contrast, an augmented ε -constraint against a normalized normal-constraint scalarization on a multi-objective mixed-integer design problem, is examined by Amirian et al. [5]. The second choice, to approximate the front rather than solve exactly, follows the precedent of Hashemi Borzabadi et al. [18], who recover Pareto fronts of multi-objective optimal control problems by evolutionary search where exact treatment is unavailable.

A chromosome is the pair (g, α) with $g \in \{0, 1, \dots, |\mathcal{S}|\}^{n_c}$ encoding cell activation and species jointly, and $\alpha \in [\alpha_{\min}, 1]^3$. Uniform crossover acts on g , blend crossover and Gaussian mutation on α .

Repair. The operator restores feasibility in three passes. (a) *Admissibility.* Every pair (p, s) violating the mask induced by (C2)–(C4) is cleared; this pass is deterministic and idempotent. (b) *Crown spacing.* Conflicting pairs under (C6) are resolved by clearing one member. Only transverse neighbours can conflict for the cell geometry used here: any two cells of the same zone that are not in the same station and adjacent columns are separated by at least $\min\{\Delta, 2\delta\}$, while $\tilde{r}_p + \tilde{r}_{p'} \leq 2 \max_s r_s < \min\{\Delta, 2\delta\}$, so (C6) cannot bind for them. The conflict graph is therefore a disjoint union of paths along each station, and the pass sweeps stations in index order and columns in increasing offset, which fixes the scan order completely. Within a conflicting pair the victim is drawn uniformly at random, independently of species and offset, and the sweep is iterated to a fixed point. (c) *Screening.* Any empty run of G median stations receives the screening species at a station drawn uniformly from the run.

The only stochastic element is the choice of victim in pass (b), and it is deliberate: any deterministic tie-break must rank the two species, and every such ranking encodes a preference among f_1, f_3, f_4 , which would bias the front the search is meant to approximate. To quantify the cost of that choice we also implemented the deterministic alternative that keeps the cell of higher canopy area per unit water, $\pi r_s^2/\omega_s$, breaking remaining ties by larger offset; Section 8.6 reports both. Because repair is applied to every offspring, all evaluated individuals satisfy (C1)–(C8) exactly and only the budget is handled by domination.

Cost and stopping. One objective evaluation costs $O(n_c|\mathcal{T}|)$ for f_1 , $O(n_c|\mathcal{R}||\mathcal{J}|)$ for f_2 once the chords of Remark 1 are cached, and $O(n_c)$ for f_3, f_4 ; repair costs $O(n_c)$ per sweep. Front quality is measured by the hypervolume indicator [34]: with the box

$$B = [1500, 1632.3] \times [58, 65.04] \times [-16, 0]$$

in the coordinates (f_1, f_2, f_3) , the indicator is the volume of the subset of B dominated by the approximation set, divided by $|B|$, so that it lies in $[0, 1]$ and equals the usual unit-cube hypervolume with reference point $(1, 1, 1)$ after affine normalization. The box is fixed across budgets and repetitions so that hypervolumes are comparable. The search is stopped when the relative hypervolume gain over a sliding window of 100 generations falls below 1%, subject to a cap of 1200 generations. A random-search control drawing the same number of repaired individuals is run at every budget.

8 Numerical Study

8.1 Instance

The instance parameters are listed in Table 3, and the species catalogue in Table 4. We instantiate (P) on a 1 km divided highway at latitude 33.3°N with azimuth $\theta = 75^\circ$, on the design day 21 July, for which the clear-sky beam exposure is $E = 24.31 \text{ MJ m}^{-2}$. The instance is representative rather than a survey of an existing road: the latitude, the azimuth and the cross-sectional dimensions are those of a standard hot-arid divided highway, and no particular corridor is being modeled. The azimuth is nevertheless consequential for Sections 8.4 and 9, since it fixes which verge can shade the travel way; by Corollary 1 it enters the model as an explicit parameter, so those results carry over to a named corridor on substituting its own θ and ϕ . The cross-section carries a 12 m median with a cable barrier on the centerline, two 14.6 m carriageways and 30 m verges; the planting domain is partitioned into $\Delta \times \delta = 20 \times 6 \text{ m}$ cells, giving $N = 50$ stations and $n_c = 550$ cells—one median column at $y = 0$ and five columns per verge at offsets 3, 9, 15, 21, 27 m—hence a combinatorial search space of 5^{550} before repair. The alignment is tangent, so (C5) is vacuous. Four receivers sit at $y = 55 \text{ m}$ on the $+$ side, which is also the

upwind side. Each carriageway is represented by a line source on its own axis, discretized into $\ell_{seg} = 20$ m segments, with source height $h_{src} = 0.5$ m and receiver height $h_{rec} = 4$ m.

The admissibility mask of (C2)–(C4) leaves 1750 admissible cell–species pairs out of 2200: with $w_m/2 = 6$ m $< W_{cz} = 9$ m, and the planting column lying on the barrier line so that (C4) does not relax (C3), the median admits frangible screening material only, so a nominally wide median contributes nothing to canopy cover. By Proposition 4 the glare-screen requirement with $G = 2$ forces 25 median activations and an irrigation floor of $w_{min} = 56$ m³ yr^{−1} km^{−1}.

Table 3: Instance parameters.

Symbol	Meaning	Value	Symbol	Meaning	Value
L	corridor length	1000 m	P_e	encroachment rate	0.60 km ^{−1} yr ^{−1}
$\Delta \times \delta$	cell size	20 × 6 m	λ	lateral decay	0.18 m ^{−1}
w_m, w_c, w_v	median, carriageway, verge	12, 14.6, 30 m	v_d^2	squared departure speed	400 m ² s ^{−2}
θ	road azimuth	75°	a_r	verge deceleration	1.47 m s ^{−2}
ϕ	latitude	33.3°N	c_b, d_{ref}	frangible severity, reference diameter	0.02, 0.40 m
E	daily beam exposure	24.31 MJ m ^{−2}	c_d, ν	dust exposure, saturation	0.55, 0.90
W_{cz}	clear zone	9 m	$\bar{h}, \zeta$	belt height, shelter scale	10, 150 m
d_{crit}	frangibility threshold	0.10 m	L'_W	source power per meter	83 dB(A)
$\gamma_{pav}, \gamma_{shd}$	pavement, shaded pavement	0.82, 0.45	a_{atm}	air absorption	5 dB km ^{−1}
γ_{soil}	bare verge soil	0.72	$\kappa, \bar{A}$	foliage rate, cap	0.06 dB m ^{−1} , 10 dB
$\gamma_{can}^0, \gamma_{can}^1$	canopy, dry and transpiring	0.55, 0.20	$\bar{A}_g$	ground-effect cap	4 dB
$\tau_0, \tau_{\pm}$	irrigation cost multiplier	1.6, 1.0	G	maximum screen gap	2 stations
α_{min}	survival sufficiency	0.50	ℓ_{seg}	source segment length	20 m
w_{defl}, y_b	barrier envelope, line	3.3 m, $y = 0$	h_{gl}^-, h_{gl}^+	screen height window	1.2, 1.8 m
h_{src}, h_{rec}	source, receiver height	0.5, 4.0 m	λ_v	latent heat	2.45 MJ kg ^{−1}

Table 4: Species catalogue, derived cluster cardinality, and verification of the hypotheses of Proposition 1 (see also Remark 3). Condition (20) requires $A_s \zeta(\lambda + b) > C_s$; it holds for every species, so each has a unique interior minimizer y_s^* , and a sign change at y_s^0 exists if and only if $A_s > C_s$, which also holds for every species.

Species		Attributes						Verification				
		h_s (m)	r_s (m)	ρ_s	d_s (m)	ω_s (m ³ yr ^{−1})	n_s	Admissible zones	A_s	C_s	y_s^0 (m)	y_s^* (m)
s_1	screening shrub	1.8	1.2	0.90	0.04	0.35	8	median and both verges	0.096	0.080	1.0	19.4
s_2	small tree	6.0	3.0	0.60	0.15	1.80	3	verges, $y^{off} \geq W_{cz}$	0.253	0.178	1.9	20.4
s_3	large tree	14.0	5.0	0.45	0.45	6.50	2	verges, $y^{off} \geq W_{cz}$	1.200	0.312	7.4	25.9
s_4	palm	12.0	3.5	0.25	0.40	4.00	2	verges, $y^{off} \geq W_{cz}$	1.200	0.149	11.5	29.9

NSGA-II is run with population 200, $p_c = 0.9$, $p_m = 3/n_c$, and 5 independent seeds per budget $W \in \{700, 1400, 2800\} \text{ m}^3 \text{ yr}^{-1} \text{ km}^{-1}$, under the stopping rule of Section 7. The doing-nothing reference is $f_1 = 1632.27 \text{ GJ km}^{-1} \text{ d}^{-1}$, $f_2 = 65.04 \text{ dB(A)}$, the latter excluding barrier insertion loss in the sense of Remark 2; $f_3 = 0$; and $f_4 = 0$.

8.2 Convergence, Front Quality and Cost

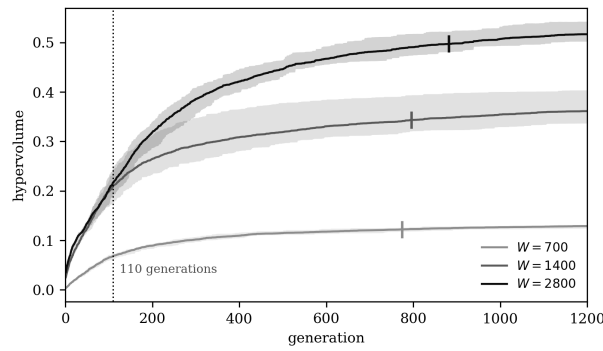


Figure 3: Hypervolume against generation, mean over 5 seeds, with the seed range shaded. The vertical marks give the generation at which the stopping rule of Section 7 first fires. A run truncated at 110 generations would have captured only 58% of the hypervolume ultimately attained at $W = 1400$.

Figure 3 shows that the search is not converged at a few hundred generations: the hypervolume is still rising steeply there, and the 1%-per-100-generation criterion is first met at generation 774, 796 and 882 for the three budgets. All results below are taken at the cap of 1200 generations, i.e. beyond the stopping generation in every case.

Table 5: Hypervolume (mean $\pm$ s.d. over 5 seeds), pooled front size, and computational cost. A realized f_4/W of 1.00 means that the irrigation budget is active (binding) at the reported design; a value below 1.00 means that the budget is slack and other constraints bind. Wall-clock times are per run, single-threaded, on one core of a 2.1 GHz Intel Xeon processor, in Python with vectorized NumPy kernels.

$W \text{ (m}^3 \text{ yr}^{-1} \text{ km}^{-1}\text{)}$	NSGA-II	Random search	Ratio	Pooled $ P $	Realized f_4/W	Evaluations	Wall-clock
700	0.1289 ± 0.0028	0.0092	14.00	114	1.00	240200	171 s
1400	0.3614 ± 0.0314	0.0553	6.53	222	1.00	240200	180 s
2800	0.5167 ± 0.0182	0.0730	7.08	569	0.84	240200	182 s

As Table 5 shows, NSGA-II attains 6.5 to 14.0 times the hypervolume of random search at every budget, with seed dispersion an order of magnitude below the gap, so the ranking is not an artifact of the seed. The cost of that advantage is 240200 objective evaluations and roughly 178 s of single-core time per run; the evaluation itself is dominated by the shading loop of (8)

and by the repair sweeps, the acoustic functional being cheap because the chords are cached (Remark 1).

The column of realized f_4/W in Table 5 records a structural transition. At $W = 700$ and $W = 1400$ the budget binds exactly; at $W = 2800$ the best designs consume only 84% of the allocation, the largest realized demand across all seeds being $2347 \text{ m}^3 \text{ yr}^{-1} \text{ km}^{-1}$. Beyond that point the active constraints are geometric—crown spacing (C6) and the clear-zone mask (C2)—(C3)—not hydrological. Additional water therefore buys nothing, which is a directly actionable conclusion for allocation policy in water-scarce jurisdictions.

8.3 Trade-Off Structure

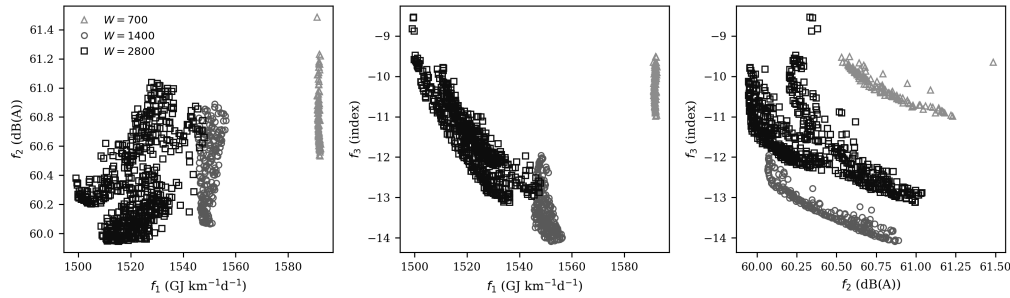


Figure 4: Pooled non-dominated sets at three irrigation budgets, in the three pairwise objective projections. Lower is better on every axis.

Raising the budget shifts the front toward lower thermal load and lower receiver level, as expected, but compresses the attainable range of f_3 : at $W = 700$ the risk index spans $[-10.99, -9.51]$, at $W = 1400$ $[-14.09, -11.96]$, and at $W = 2800$ it retreats to $[-13.12, -8.53]$. The retreat is the signature of Proposition 2(3): once the receiver-side verge saturates, the only way to keep improving f_1 is to plant the leeward verge, which adds impact hazard without adding shelter.

The three differ only marginally in f_1 and f_2 —the attainable spread is $6.5 \text{ GJ km}^{-1} \text{ d}^{-1}$ and 0.80 dB(A) —but substantially in f_3 , whose spread is 1.79 index units, and above all in species composition. The seed standard deviations in Table 6 are of the same order as the spread in f_1 and f_2 and well below it in f_3 , so the three extremes are separated by the optimizer reliably in the risk coordinate and only marginally in the other two; the composition effect below is the robust finding, the small f_1 and f_2 differences are not. The safety-leaning design carries more large-tree cells and fewer palm cells than the thermal-leaning one. This is the composition effect predicted by Remark 4: the palm delivers crown area at low water cost but combines a rigid stem with sparse foliage, so it is efficient for f_1 and inefficient for f_3 .

Table 6: Extreme designs of the pooled $W = 1400$ front. Cell counts are reported by zone in the order median / leeward / receiver, and reductions are measured against the do-nothing reference. The rows labeled “s.d.” give the standard deviation, over the 5 seeds, of the corresponding extreme design recovered independently by each seed.

Design	f_1	f_2	f_3	f_4	Cells by zone	(s_1, s_2, s_3, s_4)	Δf_1 (%)	Δf_2 (dB)
Thermal-leaning	1545.6	60.72	-13.31	1398	28 / 55 / 207	(83, 198, 8, 1)	-5.3	-4.32
s.d. over seeds	± 0.9	± 0.09	± 1.52					
Acoustic-leaning	1550.3	60.07	-12.30	1400	28 / 49 / 238	(122, 180, 9, 4)	-5.0	-4.97
s.d. over seeds	± 4.0	± 0.07	± 0.69					
Safety-leaning	1552.1	60.87	-14.09	1343	27 / 39 / 195	(67, 184, 10, 0)	-4.9	-4.17
s.d. over seeds	± 1.1	± 0.22	± 0.96					

8.4 Recovery of the Structural Predictions

Two predictions of Section 6 were not built into the search, which was given no side preference and no offset preference beyond the admissibility mask.

Asymmetry. Across the pooled fronts, the receiver-side share of activated verge cells is 0.967 on average at $W = 700$, 0.835 at $W = 1400$ and 0.699 at $W = 2800$, with an overall range of $[0.628, 1.000]$. Every design on every front exceeds one half. The decline with budget is the capacity effect of Proposition 2(3): at low budget the receiver-side verge absorbs almost all canopy, and the leeward verge is recruited only when the preferred side runs out of admissible cells or when the shading term makes leeward activation worthwhile, as Remark 5 explains.

Interior minimizer. Figure 5, right panel, shows the share of s_3 among activated receiver-side cells against lateral offset. It stays below 0.02 at every offset up to 21 m and rises to 0.18 at 27 m, the outermost available offset and the one nearest the analytic minimizer. The optimizer concentrates the hazardous species where φ_3 is most negative, converging on the neighborhood of $y_3^* = 25.9$ m without being told that this offset is special; the innermost offset of 3 m, inside the clear zone, is occupied exclusively by the frangible shrub s_1 , as (C2) requires.

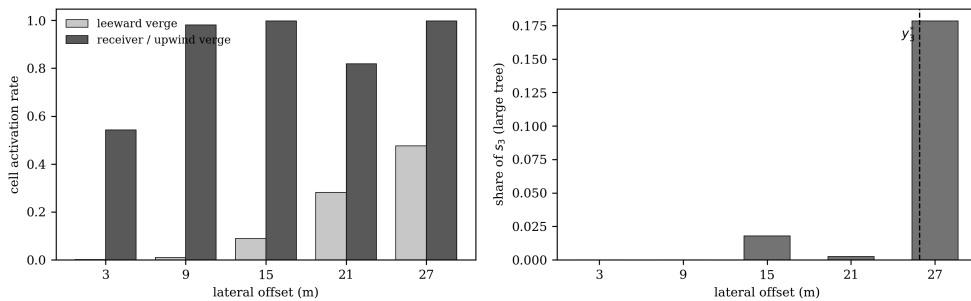


Figure 5: Left: cell activation rate by lateral offset and side, pooled over the $W = 1400$ front. Right: share of the large-trunk species s_3 among activated receiver-side cells, with the analytic minimizer y_3^* of Proposition 1 marked.

8.5 Sensitivity to the Surface-Partition Coefficients

The coefficients $\gamma_\bullet$ are reduced-form surrogates (A8), not calibrated parameters, so the thermal reductions reported above cannot be read as physical predictions without knowing how far they move with the surrogates. Table 7 perturbs each coefficient by $\pm 20\%$, recomputing both the design's f_1 and the do-nothing reference against which the reduction is measured.

Table 7: Reduction in f_1 relative to the do-nothing reference, for the thermal-leaning design of Table 6, under a $\pm 20\%$ perturbation of each partition coefficient. The unperturbed value is 5.31%.

Perturbed coefficient	γ_{pav}	γ_{shd}	γ_{soil}	γ_{can}^0	γ_{can}^1
Reduction at -20% (%)	5.71	5.31	1.14	8.54	5.33
Reduction at $+20\%$ (%)	4.96	5.30	8.53	2.07	5.29

The result is unambiguous and should temper any reading of the thermal numbers as physics. The reduction is almost insensitive to γ_{shd} and γ_{can}^1 —the first because direct pavement shading is small at this azimuth and latitude (Corollary 1), the second because the water-limited branch of (9) is active with $\Lambda = 0.019$, so $\gamma_{\text{can}} \approx \gamma_{\text{can}}^0$ throughout—but it ranges from 1.14% to 8.54% as γ_{soil} and γ_{can}^0 move within $\pm 20\%$. The *magnitude* of the thermal benefit is therefore a property of the surrogate coefficients and not a result of this study; what the model supplies is the difference between designs at fixed coefficients, and that is what the front in Figure 4 reports.

The structural conclusions are a different matter. Re-optimizing at $W = 1400$ with γ_{soil} reduced by 20%—the perturbation that moves f_1 most—leaves the receiver-side share at 0.803 against 0.835 in the unperturbed run, and leaves the ordering of species by f_3 untouched, since f_3 does not involve $\gamma_\bullet$ at all. Propositions 1 and 2 are likewise independent of the $\gamma_\bullet$: the first involves only the safety and shelter parameters, and the second only the support of the acoustic rays and the sign of the shelter increment.

8.6 Sensitivity to the Repair Rule

The randomized victim choice in pass (b) of the repair operator acts on every offspring in every generation, so its influence on the reachable part of the front is a legitimate concern. On the initial random designs the pass clears on average 29 cells per individual, and the number falls sharply as the population concentrates on sparse, spacing-feasible layouts. Table 8 compares the randomized rule with the deterministic alternative of Section 7.

The repair rule does affect front quality: the deterministic rule reaches a higher hypervolume at every budget. That is not surprising, since its priority $\pi r_s^2 / \omega_s$ is a scalarization aligned with

Table 8: Randomized versus deterministic conflict resolution in (C6): hypervolume (HV; mean $\pm$ s.d.), pooled front size $|P|$, and mean receiver-side share. The deterministic rule ranks cells by $\pi r_s^2/\omega_s$ and therefore optimizes implicitly toward f_1 and f_4 ; it is reported here as a robustness check only, and all results elsewhere in this paper use the randomized rule.

W	Randomized (reported)			Deterministic		
	HV	$ P $	Share	HV	$ P $	Share
700	0.1289 ± 0.0028	114	0.967	0.1351 ± 0.0038	96	0.959
1400	0.3614 ± 0.0314	222	0.835	0.4323 ± 0.0014	270	0.840
2800	0.5167 ± 0.0182	569	0.699	0.6069 ± 0.0110	276	0.668

f_1 and f_4 , and a repair that quietly optimizes one corner of the objective space will enlarge the dominated volume there. It is precisely for that reason that we report the neutral randomized rule: a repair operator that encodes a preference among the objectives contaminates the front it is used to approximate. What matters for the claims of this paper is that the two rules agree on the structure—the receiver-side share differs by at most 0.031 between them at every budget, and both reproduce the migration of s_3 toward y_3^* —so the structural findings of Section 8.4 are not artifacts of the stochastic pass.

9 Discussion

Three findings deserve emphasis, two of them negative. All three are statements about the behavior of this model on this instance—one latitude, one azimuth, one design day, a four-species catalogue—and we state the boundary of each. Foliage is a weak acoustic screen; the ground effect does the work. The best receiver-level reduction obtained anywhere on any front is 5.09 dB(A), and decomposing it shows where it comes from: suppressing the ground term (13) costs 3.98 dB, and suppressing the foliage term (12) costs only 1.11 dB. Almost all of the attenuation is therefore delivered by replacing an acoustically hard verge with a porous litter layer, with the ground term sitting at its cap $\bar{A}_g = 4$ dB, and hardly any by interception in the crowns. This is not a shortcoming of the optimizer: it follows from the magnitude of the foliage rate κ , of order 10^{-2} dB m $^{-1}$, and it agrees with measurements on belts of limited depth [16, 24, 31]. The claim is bounded by the belt depth and the caps: the foliage term would reach $\bar{A}$ only beyond $\bar{A}/\kappa \approx 167$ m of continuous canopy, some five times the verge depth available here, so foliage could dominate only for belts an order of magnitude deeper than a highway verge, or for κ an order of magnitude larger than measurements support. What

roadside planting supplies is a ground-cover effect that saturates quickly; the contribution here is allocation under that constraint, not magnitude.

The canopy acts as a radiation screen rather than an evaporative cooler at the irrigation rates considered. The water-limited branch of (9) is active throughout, with Λ of order 10^{-2} . Under the stated radiation model essentially all of the 2.4–8.2% thermal reduction comes from replacing bare verge soil of partition coefficient γ_{soil} with canopy of coefficient $\gamma_{\text{can}} \approx \gamma_{\text{can}}^0$, not from latent flux. Section 8.5 shows that the percentage is a property of those two surrogates, moving over 1.14–8.54% under a $\pm 20\%$ perturbation: it is the mechanism, not the number, that the model establishes. Its boundary condition is explicit, since $\Lambda \ll 1$ holds while $\lambda_v m_w \ll EA^c$; the evaporative branch would take over only for

$$W \gtrsim \frac{365 E A^c}{\lambda_v \rho_w} \approx 72865 \text{ m}^3 \text{ yr}^{-1} \text{ km}^{-1}$$

at the canopy cover realized here, about 26 times the largest budget tested in this study and far above any budget the geometry can absorb. Direct pavement shading contributes almost nothing, for the reason given in Corollary 1: at solar altitudes near $\pi/2$ the displacement $h \cot \beta$ collapses, so canopy held beyond a 9 m clear zone cannot reach the travel way during peak irradiance. The corollary also indicates when this would change: a corridor whose azimuth places the sun near $\psi - \theta = \pm \pi/2$ at peak irradiance, or a median wide enough to admit non-frangible material behind a barrier.

Wide medians can be useless. With $w_m/2 < W_{cz}$ the bilateral rule (C3) excludes every non-frangible species from the median regardless of its nominal width, so the median contributes only glare screening and consumes irrigation at the highest delivery multiplier. Median planting becomes thermally productive only when $w_m > 2W_{cz}$, or behind a barrier under (C4)—which, unlike the other two findings, is a boundary condition expressed purely in geometry and is therefore instance-independent.

10 Conclusion

Posing highway greenbelt design as a constrained multi-objective program on a discretized cross-section makes visible three properties a strip-and-setback treatment cannot express: the marginal safety contribution of a plant is non-monotone in lateral offset and attains an interior minimum well outside the clear zone, so the recoverable width is a lower bound on good practice rather than a target; the acoustic functional is structurally blind to one verge while the risk functional is not, forcing efficient designs to be transversely asymmetric; and the scalarized problem is NP-hard, with non-smooth and bilinear structure ruling out exact mixed-integer linear treatment at corridor scale. An evolutionary Pareto approximation, given no side or offset

preference, recovers both structural predictions and shows the irrigation budget ceasing to bind above roughly $2350 \text{ m}^3 \text{ yr}^{-1} \text{ km}^{-1}$, beyond which geometry, not water, limits what greening can achieve. For a design engineer the three results combine into a single ordering of decisions: fix the species first, because c_s enters f_3 quadratically in trunk diameter and no siting recovers a bad choice; then place the non-frangible part of the catalogue on the receiver-and-upwind verge at the offset y_s^* solving (21), not at the clear-zone boundary; and only then size the irrigation, which stops buying performance once the geometric constraints bind.

The limitations divide into two classes, and the distinction matters. Some are simplifications we would defend: mature-state evaluation (A2), zone-level irrigation (A4) and the tangent alignment of the instance change the numbers but not the structure. Others bear directly on the two results the paper is built around. The clear-sky beam model (A8) fixes the *magnitude* of the thermal functional through coefficients that are not calibrated, and Section 8.5 shows a $\pm 20\%$ perturbation moving the headline reduction over a factor of three; Propositions 1 and 2 are nevertheless insensitive to it, the first because φ_s contains no radiation parameter, the second because its proof uses only the support of the acoustic rays and the sign of the shelter increment, so relaxing (A8) would change where the front sits without changing either structural statement. The omission of the shading term in Proposition 2(3) is the more consequential of the two, as Remark 5 sets out: restoring it weakens the conclusion from support on one verge to bias toward it, and the size of that weakening is exactly the receiver-side shortfall below unity reported in Section 8.4. Beyond these, three extensions are natural: embedding (5) as the lower level of a bilevel program whose upper level allocates a maintenance budget over the asset life, which would restore the establishment, mortality and replacement processes excluded by (A2); replacing the expected-value accounting (A11) by a chance-constrained formulation in the manner of Chen et al. [7], so that the tail behavior dominating roadside safety is represented; and treating the azimuth θ , fixed here, as an alignment-stage decision variable, which Lemma 1 shows how to do.

Declarations

Availability of Supporting Data

All data generated or analyzed during this study are included in this published article.

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Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have influenced the work reported in this paper.

Authors' Contributions

Fatema Ali Albayati: Conceptualization; Methodology; Formal analysis; Data curation; Writing – original draft; Writing – review & editing; Supervision; Project administration; Corresponding author. **Nadia Ali Abbas Almosa:** Conceptualization; Software; Validation; Investigation; Visualization; Writing – review & editing. **Hayder F. Ghazi:** Resources; Writing – review & editing.

Artificial Intelligence Statement

Artificial intelligence (AI) tools, including large language models, were used solely for language editing and improving readability. AI tools were not used for generating ideas, performing analyzes, interpreting results, or writing the scientific content. All scientific conclusions and intellectual contributions were made exclusively by the authors.

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